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RESEARCH MEMORANDUM

A PRELIMINARY WIND-TUNNEL INVESTIGATION OF FLUTTER CHARACTERISTICS OF DELTA WINGS

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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SUMMARY

Flutter data were obtained in the Langley 4.5-foot flutter research tunnel for three delta-wing models having their leading edges swept back 45°. The tests covered a range of tunnel pressures corresponding to altitudes from sea level to 78,000 feet.

Two of the models tested fluttered over the range of pressures at values of the indicated flutter velocity and the flutter frequency which were essentially constant. Cutting off the relatively flexible tips of these models had only a minor effect upon the flutter velocity. The indicated flutter velocity and the frequency of the third model varied greatly with air density or altitude due to changes in the mode of flutter. Cutting off the tip for this case had a pronounced effect upon the flutter velocity.

The natural vibration modes of these models were found experimentally and revealed an appreciable amount of camber bending, especially at the higher frequencies.

INTRODUCTION

Delta-wing plan forms have recently gained widespread interest as high-speed plan forms and will undoubtedly attract attention from flutter analysts. Because of the comparatively recent development of the delta wing, however, very few experimental flutter data are available. To provide additional experimental data for possible corroboration of a theoretical analysis, the series of experiments reported herein was undertaken.

These experiments were carried out at tunnel pressures corresponding to an altitude range from sea level to approximately 78,000 feet in order to determine the effects of altitude upon the flutter characteristics.

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A total of 90 flutter tests were performed on three wing models having leading edges swept back 45°. Two of these models were mounted as cantilevers while the third was free to roll. One of the cantilevers was a flat plate of aluminum of constant thickness while the other two models were constructed of aluminum and balsa wood shaped to an NACA 16-004 airfoil in the stream direction.

It seemed likely that the relatively flexible tips of the models would have a significant bearing on the flutter speed; consequently, upon completion of the flutter tests of the basic wing plan forms, the tests were repeated, first with 0.7 percent of the total wing area (1/12 of the span) cut from the tips and later with 2.8 percent of the area (1/6 of the span) cut from the tips.

Because of the difficulties, as well as the uncertainties of calculating the natural vibration modes of delta wings, the modes of each model were obtained experimentally by using a photographic technique.

SYMBOLS

ρ	density of testing medium, slugs per cubic foot
ρ_o	standard density at sea level, 0.002378 slug per cubic foot
V	flutter velocity, feet per second
V_i	indicated flutter velocity, feet per second $\left(V\sqrt{\rho/\rho_o}\right)$
M	Mach number at flutter
ω	angular flutter frequency, radians per second
ω_n	natural vibration frequency of wing in the nth mode, radians per second
m	mass of wing per unit length, slugs per foot
I_{cg}	mass moment of inertia of wing section per foot of span about its center of gravity, slug-feet
l	length of semispan model measured normal to stream direction
κ	mass ratio $\left(\dfrac{\int_0^l \pi\rho b^2 dx}{m_{total}}\right)$
b	semichord of wing in stream direction

APPARATUS

Wind tunnel.- The flutter tests were made in the Langley 4.5-foot flutter research tunnel. This tunnel is of the closed-throat single-return type in which the pressure may be varied from approximately 0.5 inch of mercury absolute to atmospheric pressure.

Models.- One of the three models tested, Ia, was constructed as shown in figure 1. Model IIa was a full-span model with freedom to roll about its longitudinal axis. Construction of this model was similar to that of model Ia. The total span was 36 inches with each wing protruding 16.5 inches from a 3-inch-diameter simulated fuselage. The thickness of the 24S-T3 insert was the same as that used in model Ia, namely, 0.020 inch. Model IIIa was a flat plate of 0.102-inch 24S-T3 aluminum alloy with rounded leading and trailing edges. The plan form was identical with that of model Ia.

During the course of the flutter tests, portions of the tips were cut from all the models in order to determine the effects of the relatively flexible tips upon the flutter speed. With one-twelfth of the span cut off, the model designations were changed to Ib, IIb, and IIIb. Likewise, when one-sixth of the span was removed from the tip, the designations became Ic, IIc, and IIIc.

Four sets of strain gages were mounted on each wing as shown in figure 1 for recording the bending and torsional stresses at these points.

DETERMINATION OF MODEL PARAMETERS

Significant structural parameters that may be conveniently used for a theoretical flutter analysis of a delta wing have not been definitely established. It is generally believed that an analysis utilizing some form of influence coefficients would more accurately represent the problem than an analysis utilizing beam theory. The possibility exists, however, that use of the simpler beam theory might result in a reasonably good approximation of the flutter speed. Accordingly, some model parameters were determined that might be useful for each of the methods.

Tables I and II give the influence coefficients for models Ia and IIIa. These coefficients are given as the deflection in inches per unit load for each of the 16 stations on the wing shown in figure 2.

In connection with beam theory, two loci of flexual centers were obtained for each of the wing models. The first of these loci was located by successively applying a concentrated load at various points along a chord lying in the stream direction until a point was located which produced no twisting of this chord. This procedure was then repeated at several spanwise stations until enough points were obtained to enable the drawing of a smooth curve. The second locus of flexual centers was found in a similar manner but the chord was considered to be normal to the bisector of the tip angle. The loci of flexural centers so obtained are shown in figure 3.

Two section centers of gravity were also found for each model. The section centers of gravity in the stream direction and normal to the bisector of the tip angle for model I were at 49 percent and 48 percent of the chord, respectively, - for model II, at 48.8 percent and 49.5 percent, respectively. Both centers of gravity of the flat-plate model were, of course, at 50 percent of the chord.

The variation of mass along the span (chord parallel to the air stream) for the various models is plotted in figure 4. Also plotted is the variation of mass along the bisector of the tip angle, in which case the chord was considered to be normal to this line.

Figure 5 gives the variation of the mass moment of inertia of the wing sections along the span as well as along the bisector of the tip angle. One set of curves shows the mass moments of inertia of the sections about an axis normal to the air stream and passing through the section centers of gravity. The remaining curves are the mass moments of inertia of the section about an axis parallel to the bisector of the tip angle and passing through the section centers of gravity.

It should be pointed out that where the span is measured in the direction of the midchord, the values of center of gravity, m, and I_{cg} near the root were extrapolated to correspond to values of a fictitious plan form as shown by the dashed lines in figure 1.

The natural vibration modes of the wings were obtained experimentally by exciting each of the wings at its natural vibration frequencies and photographing the wing at the maximum amplitude of the cycle (reference 1). The resulting amplitudes are plotted in figures 6 to 9. The wings were excited by means of a calibrated electronic oscillator driving a magnetic shaker. Photographs of model II were not made. It appeared, however, that the vibration-mode shapes of this model were very similar to those of model I.

The natural vibration frequencies were recorded while photographing the natural vibration modes and were as follows:

Model	ω_1 (radians/sec)	ω_2 (radians/sec)	ω_3 (radians/sec)	ω_4 (radians/sec)	ω_5 (radians/sec)
Ia	176	358	408	710	867
Ib	179	358	440	710	917
Ic	204	364	521	710	973
IIa	410	855	1010	1561	1840
IIb	417	858	1100	1564	1950
IIc	477	867	1300	1570	2070
IIIa	28.3	113	166	----	----
IIIb	28.3	116	170	----	----
IIIc	28.3	126	176	----	----

TEST PROCEDURE

Since flutter is generally a destructive phenomenon it is necessary to exercise great care during a flutter test. The tunnel speed was, therefore, increased slowly during the runs, the increases being in smaller increments as the critical flutter speed was approached. At the critical flutter speed, the necessary tunnel data were recorded and an oscillograph record of the flutter frequency was taken. The tunnel speed was then immediately reduced in order to preserve the model. A sample oscillograph record taken at flutter is shown as figure 10.

In these tests, the tunnel was operated at pressures from approximately atmospheric pressure to 1/25 atmosphere, corresponding to an altitude range from sea level to approximately 78,000 feet, and at Mach numbers up to 0.81. The Reynolds number at flutter, based on the chord at midsemispan, varied from 0.33×10^6 to 4.19×10^6.

Measurements taken during the tests included static pressure, dynamic pressure, temperature, and the flutter frequency. From these data the density of the testing medium, flutter velocity, mass-ratio parameter, indicated flutter velocity, flutter frequency, Mach number, Reynolds number, and equivalent standard density altitude were computed. These parameters are compiled in tables III, IV, and V.

In order to ascertain whether the models had been damaged or weakened by flutter, oscillograph records were taken before and after each run at zero airspeed with the models excited at their first and second natural frequencies. These frequencies did not change a measurable amount throughout the series of tests.

RESULTS AND DISCUSSION

Results of flutter tests are given in figures 11, 12, and 13 in which the indicated flutter velocity V_i, as well as the circular flutter frequency ω, is plotted against standard density altitude.

Effects of altitude.- As can be seen from figure 11, both the indicated flutter velocity and the flutter frequency of the flat-plate models are nearly constant over the range of altitudes from sea level to approximately 50,000 feet. Above 50,000 feet there is a gradual decrease in both the indicated flutter velocity and flutter frequency. This drop is probably due to the relatively high Mach numbers obtained in this range, $M = 0.55$ to 0.80. The absence of any abrupt change in the flutter velocity and frequency indicated there was no change in the mode of flutter.

Unlike the flat-plate models, the results in figure 12 show that models Ia and Ic fluttered in three different modes over the range of altitudes from sea level to 78,000 feet while model Ib fluttered in four different modes. These changes in mode were obvious to the observer of the flutter tests and are shown in figure 12 wherever there is a change in the slope of the flutter-speed curves and an abrupt change in the frequency.

It is interesting to note in figure 12 that at an altitude of 50,000 feet model Ib fluttered in two different modes. This phenomenon was possible because flutter at the lower velocity was sufficiently mild that this flutter speed could be exceeded without damage to the model.

Also in figure 12 are eight flutter points obtained from a model (I') identical to model I. Considering the many changes in the flutter mode, this model fluttered at velocities which were surprisingly close to those of model I. Due to malfunctioning of the recording oscillograph the flutter frequencies of model I' were not obtained. Model I' was inadvertently fluttered to destruction at a velocity of 433 feet per second and a pressure altitude of 13,6000 feet. The flutter parameters of this model are included in table V.

Although the wing construction of the rolling model, II, was quite similar to that of model I, the flutter characteristics (fig. 13) differed considerably. Model II displayed none of the many mode changes with altitude characteristic of model I but fluttered at an indicated flutter velocity and frequency which were relatively constant over the entire range of altitudes. Flutter of the rolling model was in a symmetric mode.

Effect of cutting off tip.- As can be seen from figures 11 and 13, cutting off a portion of the wing tips of models III and II had only small effects on the indicated flutter velocities and frequencies. Model I, however, reacted somewhat differently. At sea level, (fig. 12) the relative flutter speeds of the three models (Ia, Ib, and Ic) were as would be expected; that is, model Ia, whose tip was quite flexible, fluttered at the lowest speed, while model Ic, which had one-sixth of the span cut from the tip, fluttered at the highest speed. The relative flutter velocities of these three models change with altitude until at 50,000 feet the order is the reverse of that at sea level.

Remarks on theoretical considerations.- Some indications as to the problems of performing a theoretical analysis for a delta-wing configuration can be obtained from an examination of the experimental results. A comparison of the frequencies at which the models fluttered (figs. 11, 12, and 13) with the frequencies of the natural vibration modes given previously shows that models III and II fluttered at frequencies which lay between those of the first and second natural vibration modes. Model I, however, fluttered at several distinctly different frequencies which fell at random between the frequencies of the first and fourth natural modes of the model. Thus a modal type of flutter analysis of a delta wing would probably require four or more degrees of freedom. Examination of the mode shapes of the three models at the higher frequencies (figs. 6 to 9), however, shows large distortions of the airfoil camber line at these frequencies. It would thus seem that appreciable error might be introduced by the use of a structural representation of wings of very low aspect ratio which considers only the deflections of the locus of flexural centers. A more appropriate approach to the problem which could account in some degree for the camber bending would be the utilization of influence coefficients at several chordwise and spanwise positions such as those given for the present models in tables I and II.

Although, as discussed previously, a modal analysis neglects some factors, such an analysis of the Raleigh-Ritz type was made for model I to determine if the error would be important. The calculations were made with the assumption of two-dimensional air flow and three degrees of freedom. It is realized that the use of two-dimensional air forces is not realistic for such low-aspect-ratio wings and certainly contributed to the fact that the calculated flutter velocities were only 40 to 60 percent of the experimental values. These results were obtained with the assumption of the chord to be parallel to the air stream. The error was reduced only slightly by assuming the chord to be normal to the bisector of the tip angle. On the basis of these results, it appears that a theoretical flutter analysis of a delta wing should take into account the camber bending found at the higher frequencies and should utilize three-dimensional air forces.

CONCLUDING REMARKS

The results of 90 wind-tunnel flutter tests carried out on three delta-wing models for a range of tunnel pressures corresponding to altitudes from sea level to 78,000 feet have been presented. Two of the models tested fluttered over the range of pressures at values of the indicated flutter velocity and the flutter frequency which were essentially constant. Cutting off the relatively flexible tips of these models had only a minor effect upon the flutter velocity. The indicated flutter velocity and the frequency of the third model varied greatly with the altitude or air density due to changes in the mode of flutter. Cutting off the tip for this case had a pronounced effect upon the flutter velocity.

Comparison of the flutter frequencies of the models with the frequencies of the natural modes of vibration showed that one of the models fluttered at several distinctly different frequencies which fell at random between the frequencies of the first and fourth natural modes of this model. Thus, it appears that a theoretical analysis for this type of configuration should be able to represent at least the mode shapes of the first four degrees of freedom. Further, since photographs of the models vibrating at their natural frequencies showed large distortion of the camber line at the higher frequencies, the method of analysis should be able to represent the mode shapes in the chordwise as well as the spanwise directions.

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REFERENCE

1. Herr, Robert W.: Preliminary Experimental Investigation of Flutter Characteristics of M and W Wings. NACA RM L51E31, 1951.

TABLE I.- INFLUENCE COEFFICIENTS FOR MODEL Ia

[Values given in (in./lb)10^{-3}]

Station	A1	B1	C1	A2	B2	C2	A3	B3	C3	A4	B4	C4	A5	B5	C5	B6
A1	9.40															
B1	.21	1.60														
C1	0	.15	5.60													
A2	7.60	1.20	.01	26.0												
B2	.70	2.70	.44	5.00	5.2											
C2	0	1.73	3.50	.35	5.0	17.0										
A3	4.20	2.45	.17	24.0	11.2	2.9	68.0									
B3	1.50	4.00	.86	10.0	10.0	8.1	28.0	26.0								
C3	.23	3.30	1.70	2.50	10.8	21.0	12.0	28.0	50.0							
A4	3.40	3.50	.48	18.0	15.8	6.8	65.0	46.5	25.0	142						
B4	2.15	4.40	1.11	14.0	14.0	10.5	46.5	40.0	38.5	83	89					
C4	1.24	4.00	1.60	7.70	17.0	15.0	30.0	47.0	59.0	62	83	116				
A5	3.30	4.00	.76	15.5	18.5	10.2	62.0	58.0	36.5	130	111	91	270			
B5	2.65	4.60	1.21	16.5	16.6	11.5	54.0	53.0	45.0	107	115	106	205	225		
C5	1.60	5.40	1.50	14.5	16.2	11.5	49.0	51.0	60.0	79	113	143	160	185	240	
B6	3.00	4.60	1.20	16.0	19.0	12.5	60.0	62.0	47.0	130	135	116	310	265	260	510

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TABLE II.- INFLUENCE COEFFICIENTS FOR MODEL IIIa

[Values given in (in./lb)10^3]

Station	A1	B1	C1	A2	B2	C2	A3	B3	C3	A4	B4	C4	A5	B5	C5	B6
A1	7.52															
B1	3.22	6.32														
C1	1.58	3.97	10.2													
A2	11.6	14.1	12.9	58.2												
B2	8.19	13.4	17.5	53.6	63.4											
C2	6.20	12.1	22.8	49.0	67.5	87										
A3	12.8	19.4	24.8	83.5	97.1	107	179									
B3	11.1	17.7	26.7	77.9	97.8	118	186	197								
C3	9.99	17.2	29.4	73.5	98.5	129	190	210	233							
A4	13.1	20.7	31.0	86.5	115	138	217	241	259	324						
B4	11.7	19.5	30.5	85.9	110	138	208	237	256	331	331					
C4	10.9	18.8	31.5	83.3	114	145	212	245	276	345	344	376				
A5	13.0	20.5	32.8	94.0	120	150	232	263	284	362	376	396	442			
B5	12.5	21.0	33.9	93.2	119	152	232	264	288	363	376	400	453	453		
C5	12.6	19.8	33.1	90.1	116	149	228	265	291	357	384	404	470	454	480	
B6	12.2	20.4	33.3	91.9	119	153	235	267	296	367	391	415	468	481	502	539

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TABLE III. EXPERIMENTAL DATA FOR MODEL III

Model	ρ (slugs/cu ft)	$\sqrt{1/\kappa}$	V (fps)	ω (radians/sec)	Mach number	V_i (fps)	Test Reynolds number	Standard density altitude (ft)
IIIa	0.002408	3.54	252	94.8	0.229	254	2.45×10^6	0
	.001980	3.90	267	95.4	.241	243	2.13	6,100
	.001541	4.42	296	94.8	.268	238	1.84	14,100
	.001156	5.10	337	93.0	.305	235	1.57	22,700
	.000927	5.70	380	91.1	.345	238	1.42	28,900
	.000578	7.22	481	93.0	.438	238	1.12	40,200
	.000365	9.07	596	78.5	.545	233	.878	49,800
	.000239	11.22	716	79.8	.660	227	.691	59,000
	.000165	13.47	793	64.1	.735	208	.528	66,400
	.000144	14.45	812	57.8	.750	199	.472	69,200
IIIb	.002324	3.58	249	94.2	.223	246	2.34	800
	.001916	3.95	268	94.2	.240	241	2.07	7,200
	.001522	4.43	309	91.1	.278	248	1.90	14,500
	.001115	5.19	365	96.7	.324	249	1.64	23,700
	.000619	6.95	483	88.0	.437	246	1.21	38,700
	.000460	8.06	548	84.0	.498	241	1.02	45,000
	.000341	9.36	628	81.6	.573	238	.865	51,000
	.000251	10.95	723	75.0	.665	235	.733	58,000
	.000163	13.55	824	63.0	.767	216	.542	66,600
IIIc	.002324	3.54	258	99.2	.232	255	2.42	800
	.001920	3.90	283	98.6	.255	254	2.19	7,100
	.001530	4.36	310	97.4	.279	249	1.91	14,300
	.001141	5.06	359	95.5	.324	249	1.65	23,100
	.000764	6.19	440	95.5	.399	249	1.36	34,000
	.000580	7.10	504	89.2	.458	249	1.18	40,100
	.000419	8.35	588	89.2	.536	246	.995	46,900
	.000260	10.60	726	86.6	.668	241	.762	57,200
	.000185	12.60	830	67.2	.769	232	.537	64,200
	.000154	13.80	863	67.0	.800	220	.537	68,000

TABLE IV. EXPERIMENTAL DATA FOR MODEL I

Model	ρ (slugs/cu ft)	$\sqrt{1/\mu}$	V (fps)	ω (radians/sec)	Mach number	V_i (fps)	Test Reynolds number	Standard density altitude (ft)
Ia	0.002319	2.60	334	289	0.301	330	3.13×10^6	800
	.001654	3.08	424	295	.387	353	2.83	11,800
	.001159	3.68	514	622	.470	358	2.40	22,800
	.000837	4.33	589	609	.540	351	1.99	31,700
	.000592	5.15	698	603	.640	349	1.67	39,700
	.000455	5.87	753	446	.690	330	1.38	45,200
	.000324	6.97	763	433	.700	282	.998	52,300
	.000285	7.43	782	421	.720	271	.899	55,000
	.000216	8.53	804	433	.740	242	.701	60,700
	.000181	9.32	806	433	.740	223	.589	64,000
	.000154	10.08	833	421	.770	213	.518	67,500
Ib	.002203	2.66	380	289	.336	367	3.38	2,600
	.001832	2.93	378	653	.334	331	2.80	8,600
	.001439	3.31	398	647	.352	309	2.31	16,200
	.001070	3.83	433	659	.384	290	1.87	25,000
	.000770	4.51	483	665	.429	276	1.50	33,700
	.000619	5.03	512	665	.454	263	1.28	38,800
	.000510	5.54	547	659	.486	254	1.13	42,800
	.000432	6.02	588	659	.523	251	1.03	46,200
	.000368	6.52	618	659	.550	243	.919	49,500
	.000291	7.34	699	647	.620	243	.822	54,100
	.000365	6.55	670	502	.626	263	.988	49,800
	.000314	7.06	673	496	.630	245	.855	52,900
	.000243	8.03	692	496	.647	221	.679	58,150
	.000209	8.66	696	490	.651	207	.588	61,250
	.000136	10.74	743	483	.694	177	.408	70,000
	.000095	12.84	859	368	.810	172	.330	77,500
Ic	.002053	2.74	506	628	.451	471	4.19	5,000
	.001902	2.84	525	628	.469	469	4.03	7,400
	.001652	3.05	559	628	.502	466	3.11	12,000
	.001305	3.44	567	546	.512	419	2.99	19,200
	.000990	3.95	581	546	.522	375	2.32	27,200
	.000763	4.50	573	546	.516	326	1.77	33,900
	.000627	4.96	577	546	.521	296	1.46	38,400
	.000481	5.66	589	553	.534	265	1.14	44,100
	.000390	6.28	602	559	.547	243	.947	51,400
	.000316	7.00	619	565	.564	226	.790	53,100
	.000261	7.69	650	571	.596	216	.685	56,900
	.000198	8.03	664	559	.608	192	.531	62,500
	.000125	11.10	829	383	.777	191	.418	72,000

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TABLE V.- EXPERIMENTAL DATA FOR MODEL II AND I'

Model	ρ (slugs/cu ft)	$\sqrt{1/k}$	V (fps)	ω (radians/sec)	Mach number	V_i (fps)	Test Reynolds number	Standard density altitude (ft)
IIa	0.002105	3.61	406	754	0.355	381	1.575×10^6	4,100
	.001770	3.93	434	729	.380	374	1.41	9,800
	.001363	4.48	478	710	.419	362	1.20	17,900
	.001027	5.17	540	716	.474	355	1.02	26,000
	.000646	6.51	623	685	.551	326	.74	37,800
	.000460	7.72	696	672	.619	307	.588	44,800
	.000317	9.31	784	660	.702	286	.457	52,800
IIb	.002250	3.48	406	760	.362	395	1.83	1,900
	.001833	3.84	443	754	.394	389	1.63	8,600
	.001429	4.35	484	748	.430	375	1.39	16,500
	.001039	5.10	551	735	.491	364	1.15	25,600
	.000861	5.62	591	729	.526	356	1.02	30,800
	.000668	6.37	648	723	.576	343	.867	37,000
	.000480	7.50	730	710	.650	329	.703	44,000
	.000361	8.66	798	691	.714	311	.578	49,800
IIc	.002121	3.53	399	823	.348	377	1.84	3,900
	.001696	3.95	447	817	.390	377	1.65	11,100
	.001347	4.42	492	810	.430	370	1.44	18,300
	.000985	5.19	570	798	.501	367	1.22	27,100
	.000630	6.46	701	792	.622	361	.959	38,200
	.000448	7.67	811	785	.728	352	.790	45,400
Ia'	.002340	2.60	374	?	.340	371	3.52	500
	.001651	3.09	429		.390	358	2.85	12,000
	.001156	3.69	504		.458	352	2.34	22,700
	.000841	4.33	607		.553	361	2.05	31,400
	.000597	5.17	718		.658	359	1.72	39,500
	.000381	6.43	791		.732	317	1.21	48,600
Ic'	.002233	2.64	493		.445	477	5.17	2,100
	.001569	3.15	532		.482	433	3.91	13,600

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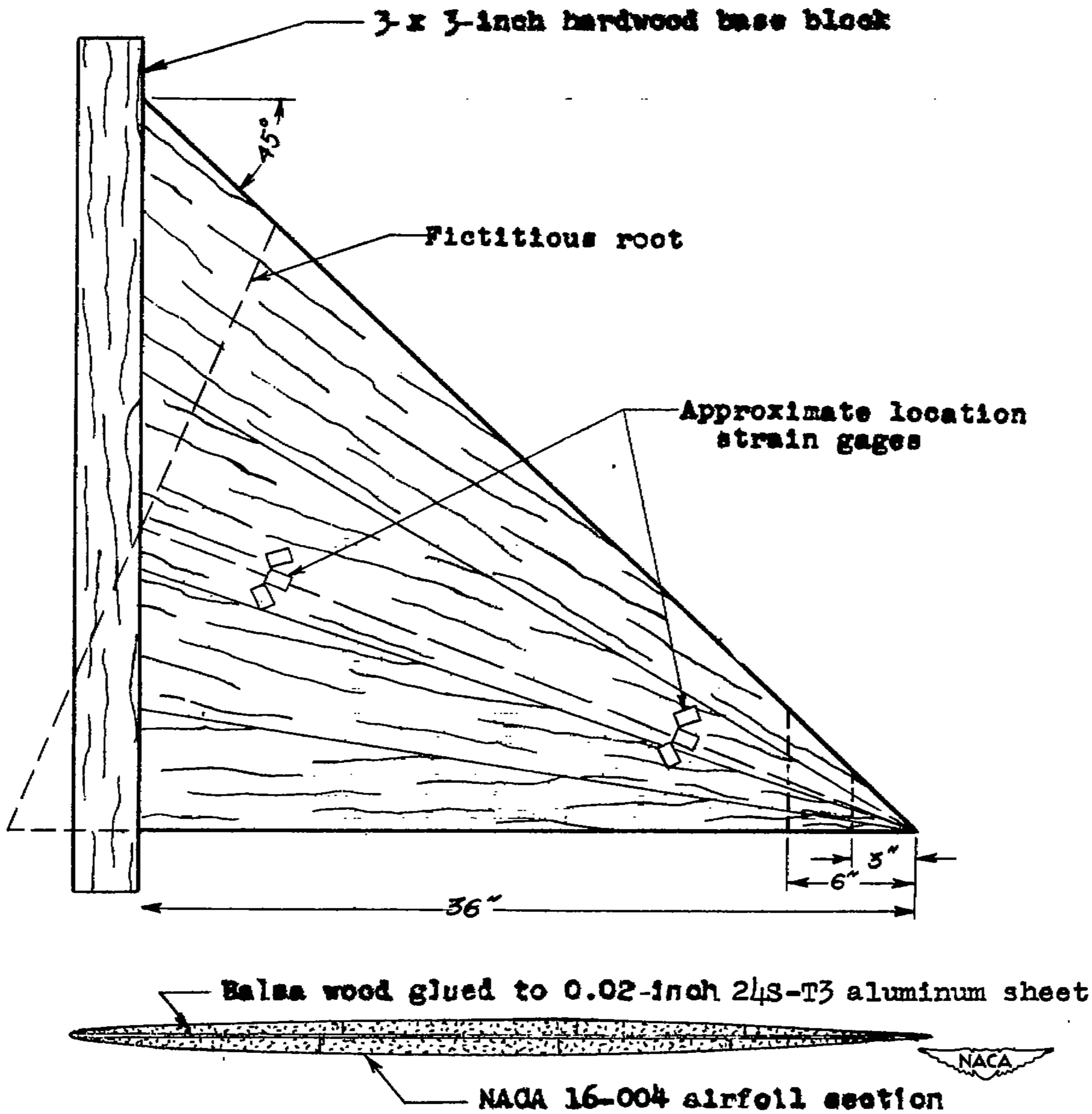


Figure 1.- Plan and cross-sectional views of model Ia.

[redacted]

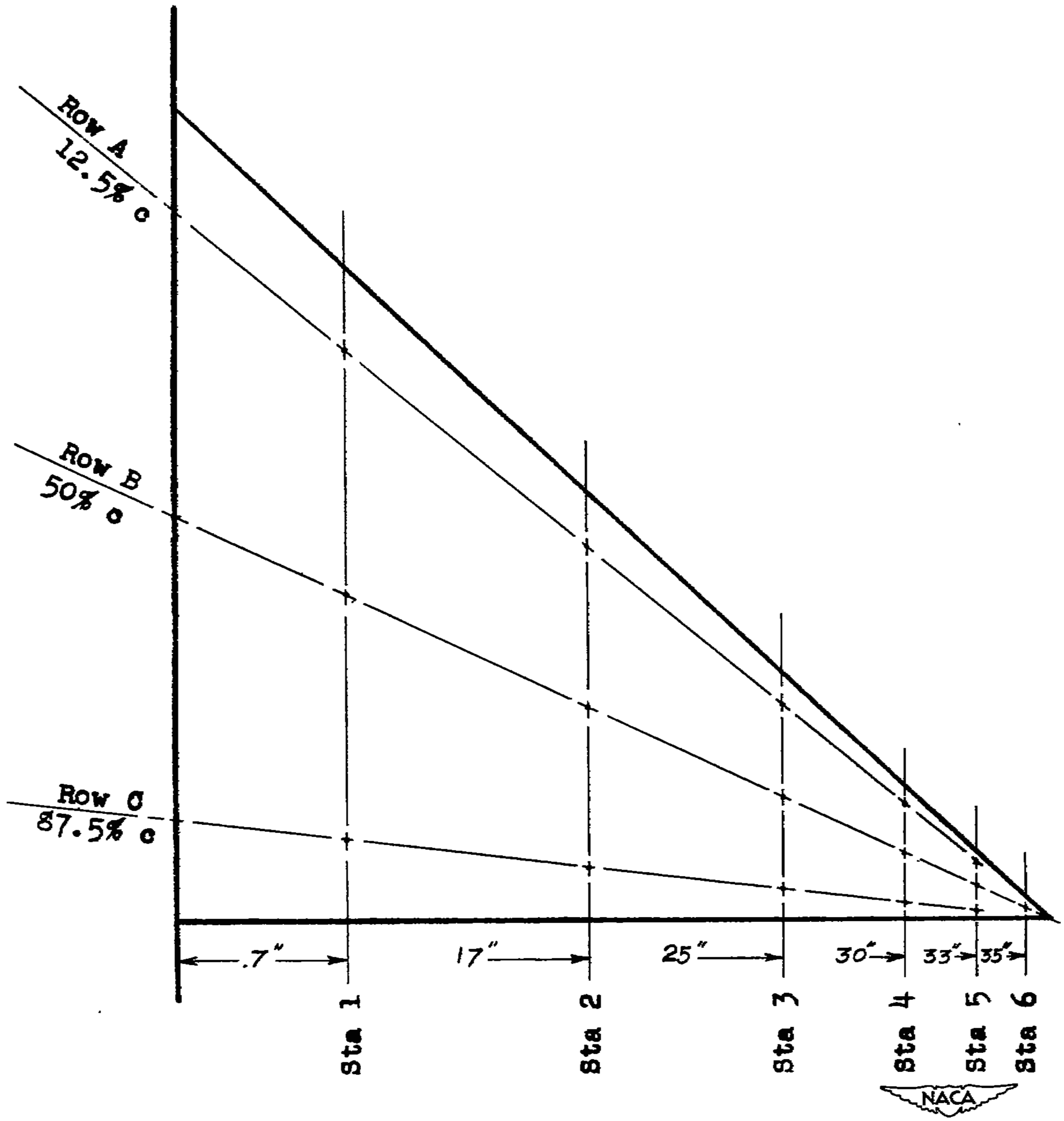


Figure 2.- Location of points at which influence coefficients were measured.

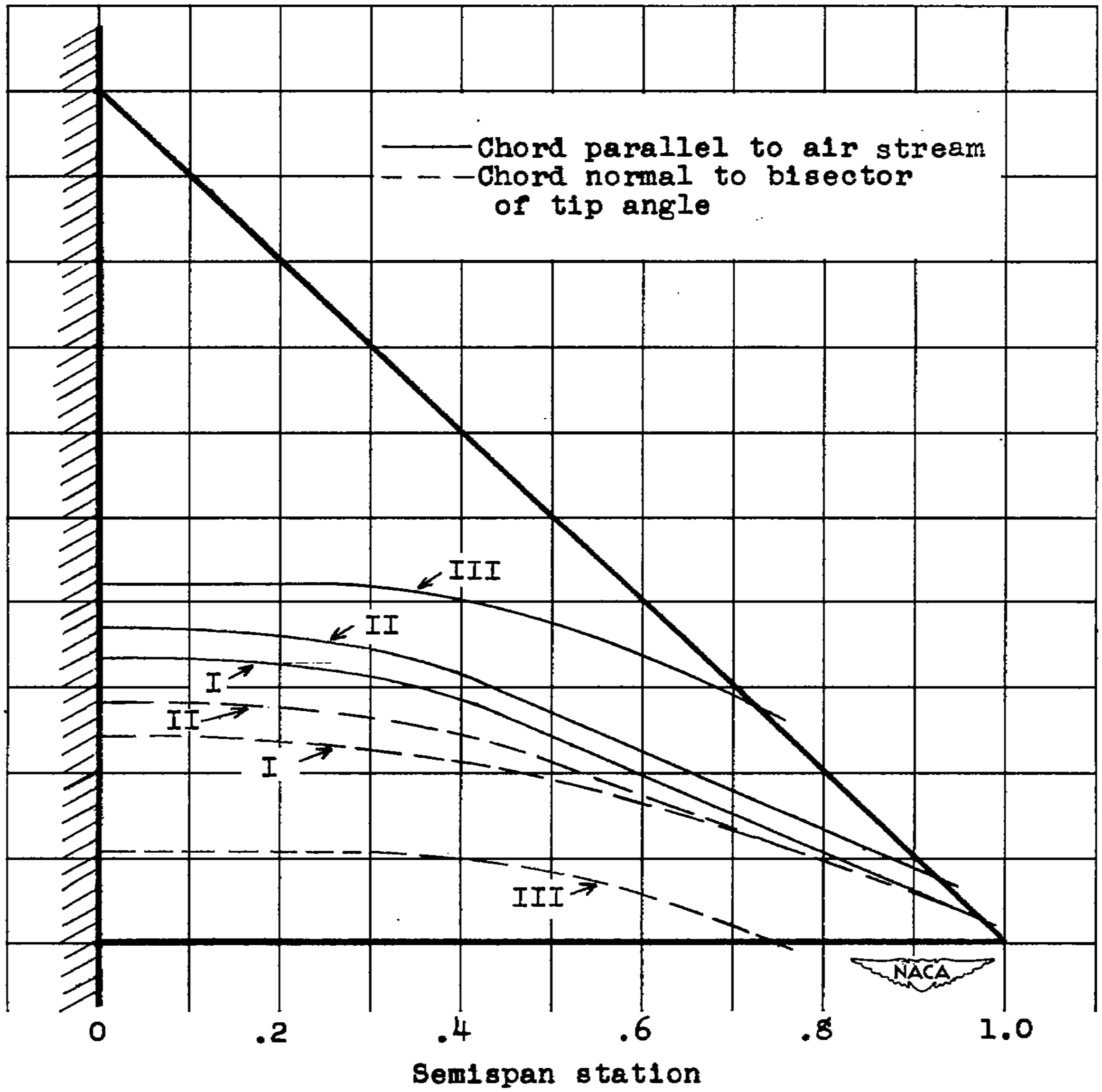


Figure 3.- Loci of flexural centers.

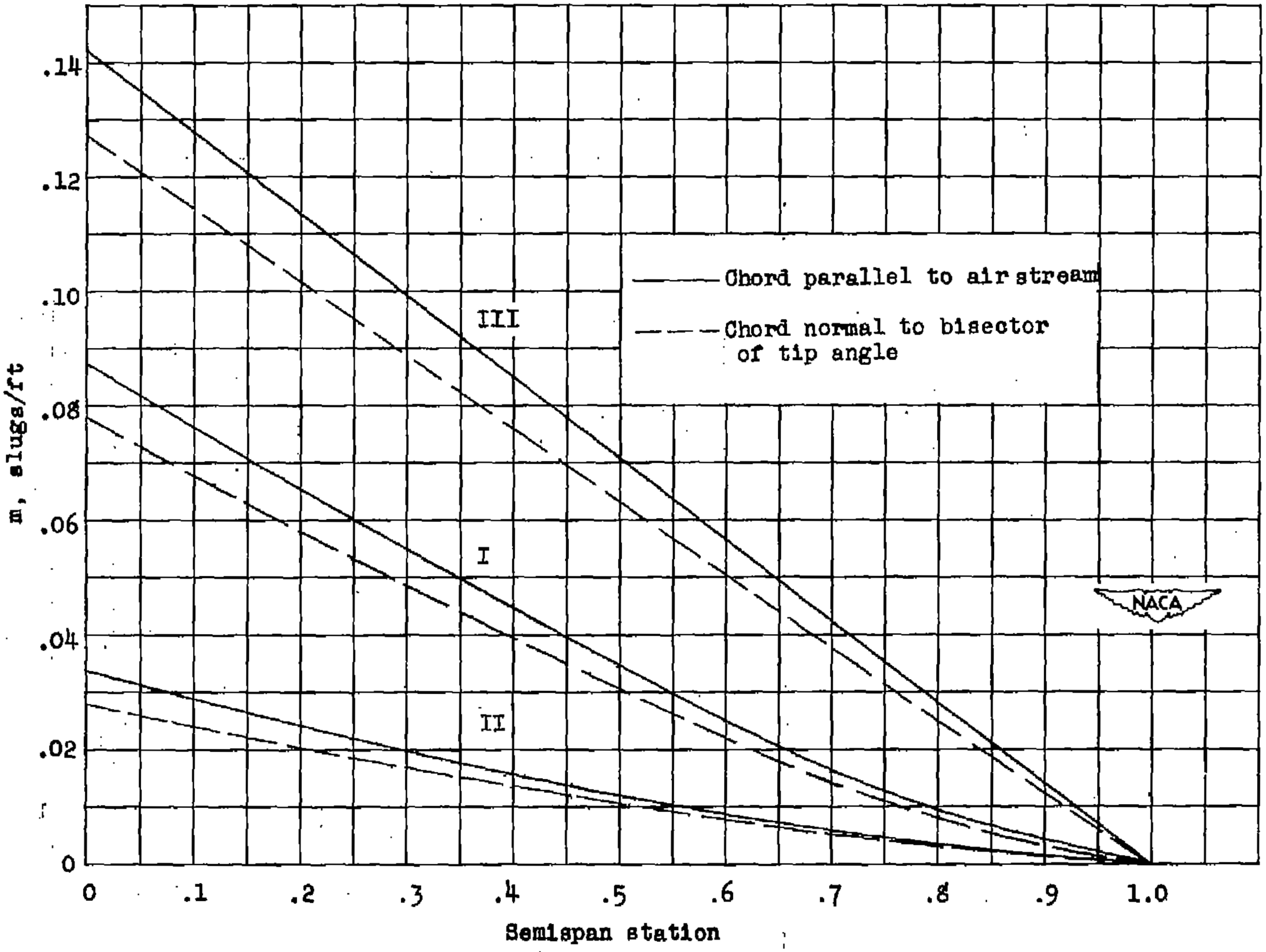


Figure 4.- Variation of mass m with span.

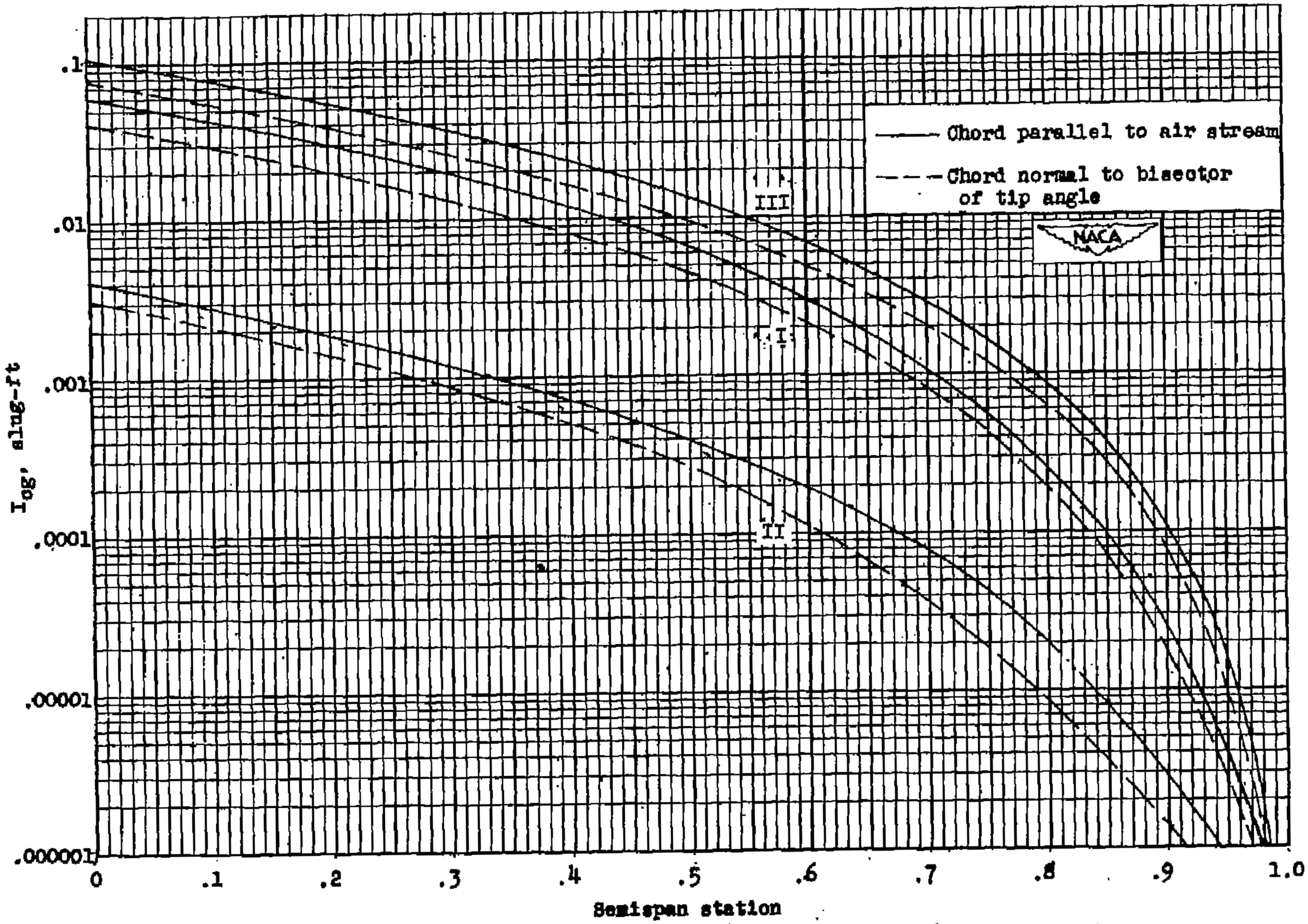
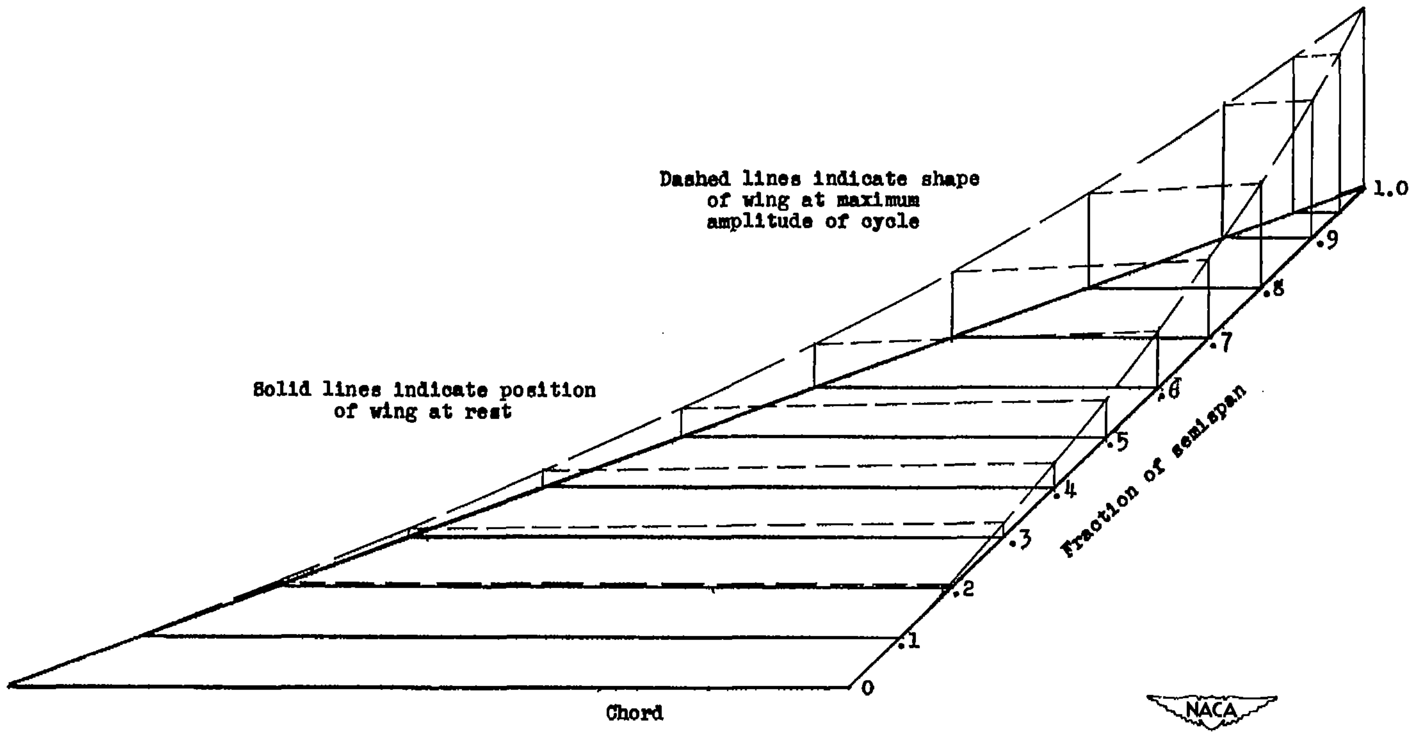
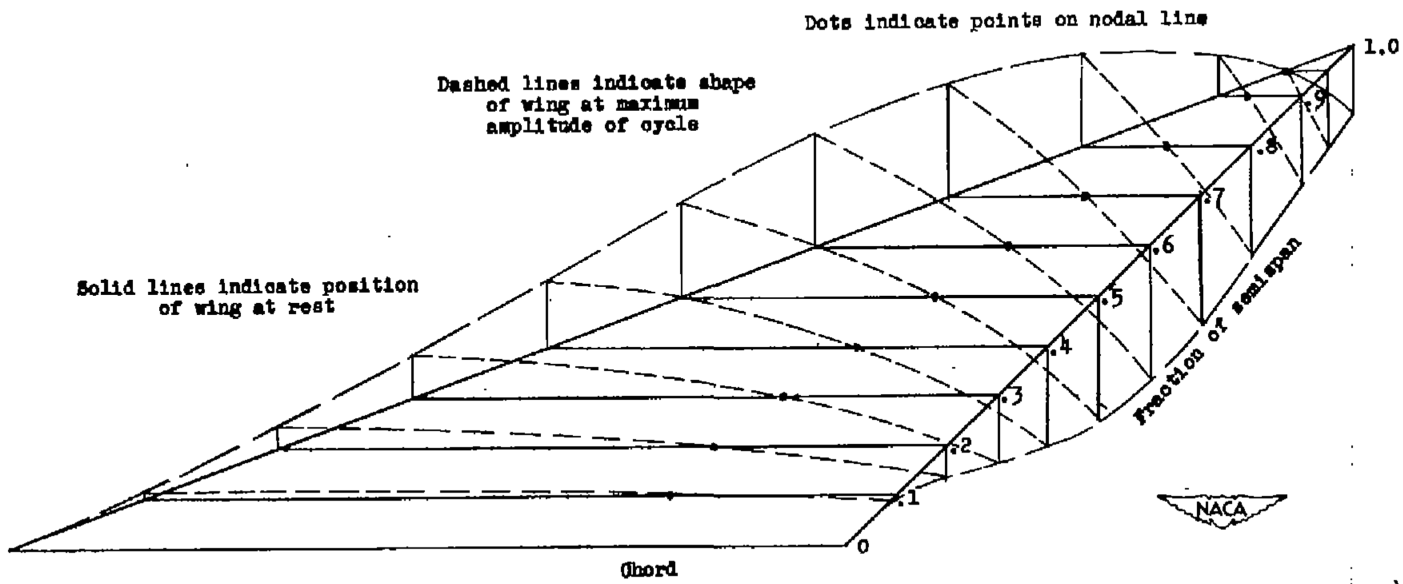


Figure 5.- Variation of mass moment of inertia I_{cg} with span.



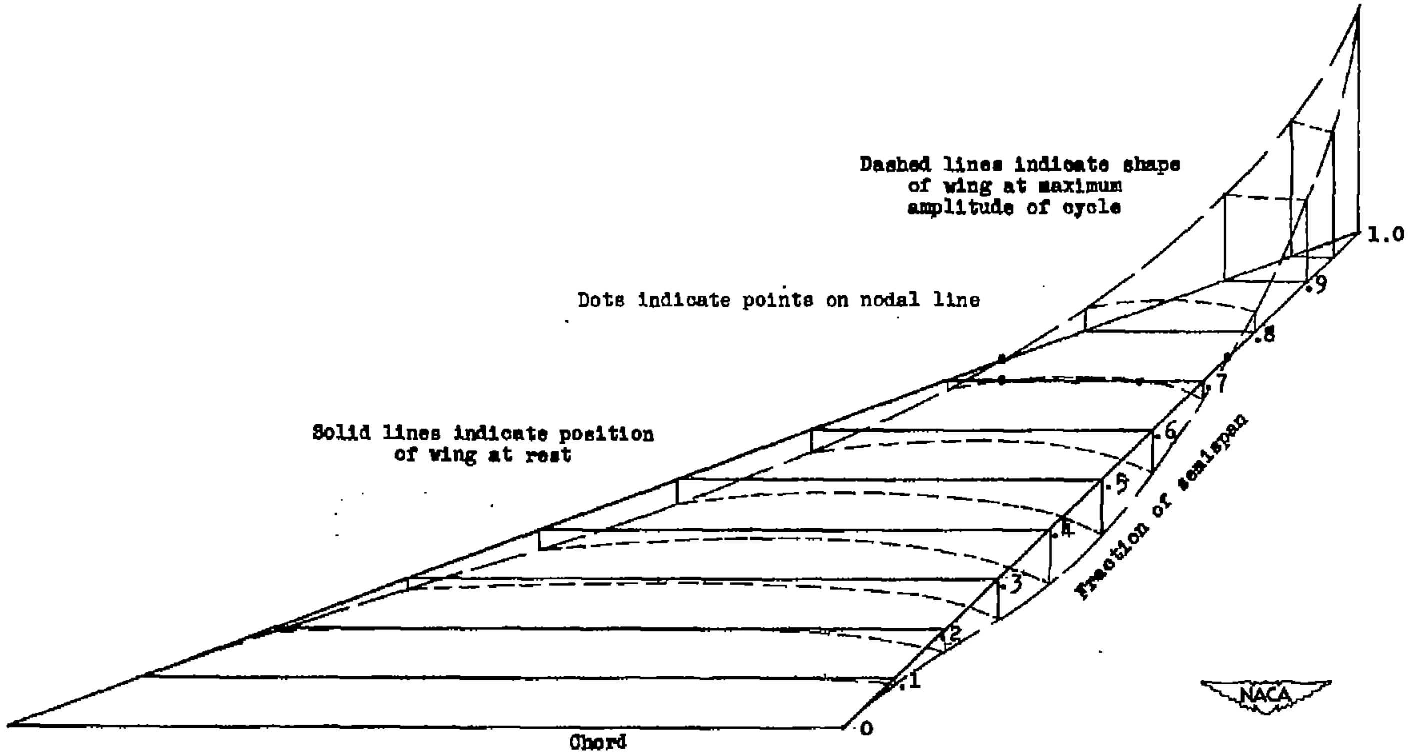
(a) First natural vibration mode, $\omega_1 = 176$ radians per second.

Figure 6.- Natural vibration modes of model Ia.



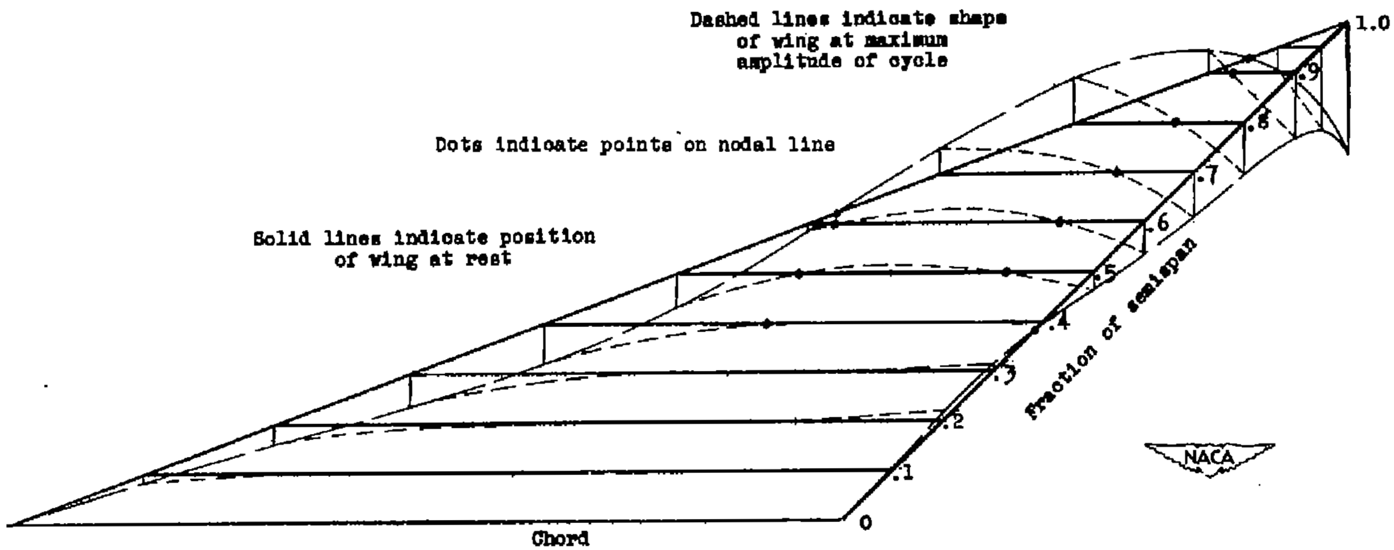
(b) Second natural vibration mode, $\omega_2 = 358$ radians per second.

Figure 6.- Continued.



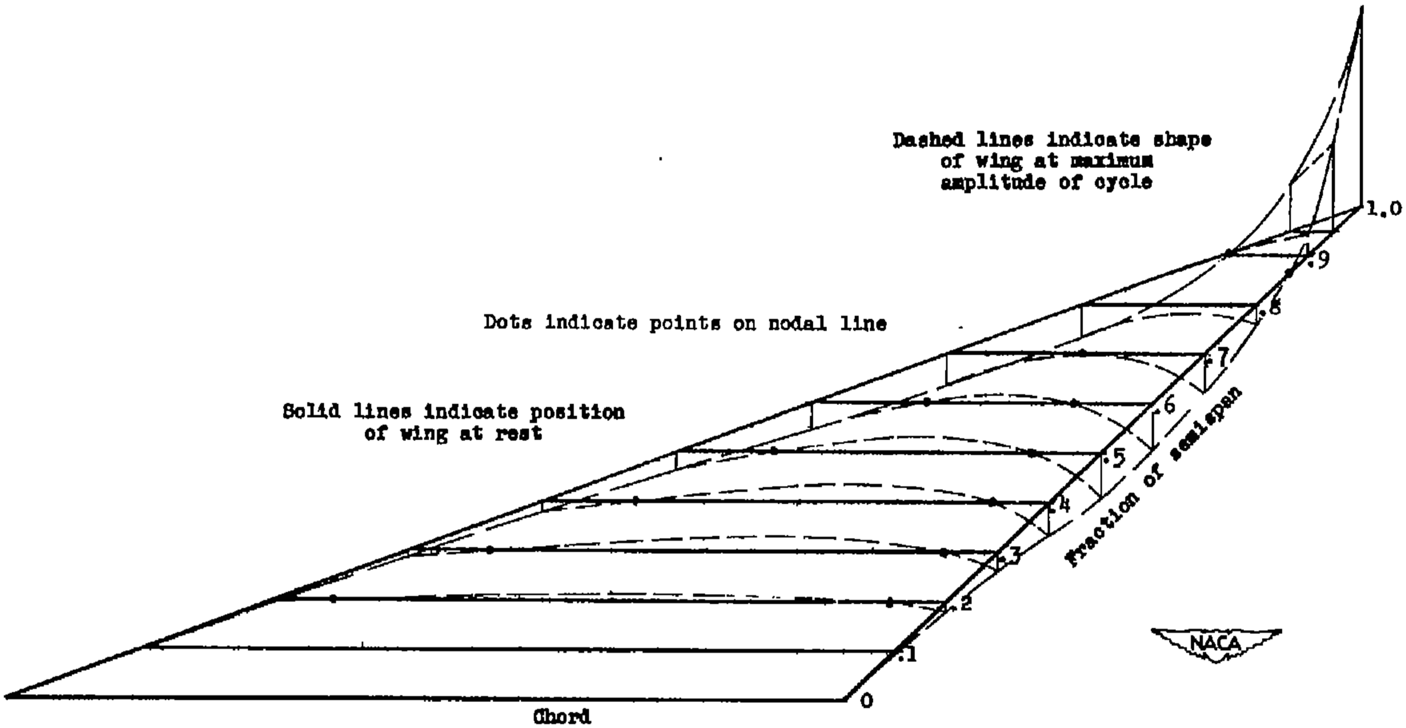
(c) Third natural vibration mode, $\omega_3 = 408$ radians per second.

Figure 6.- Continued.



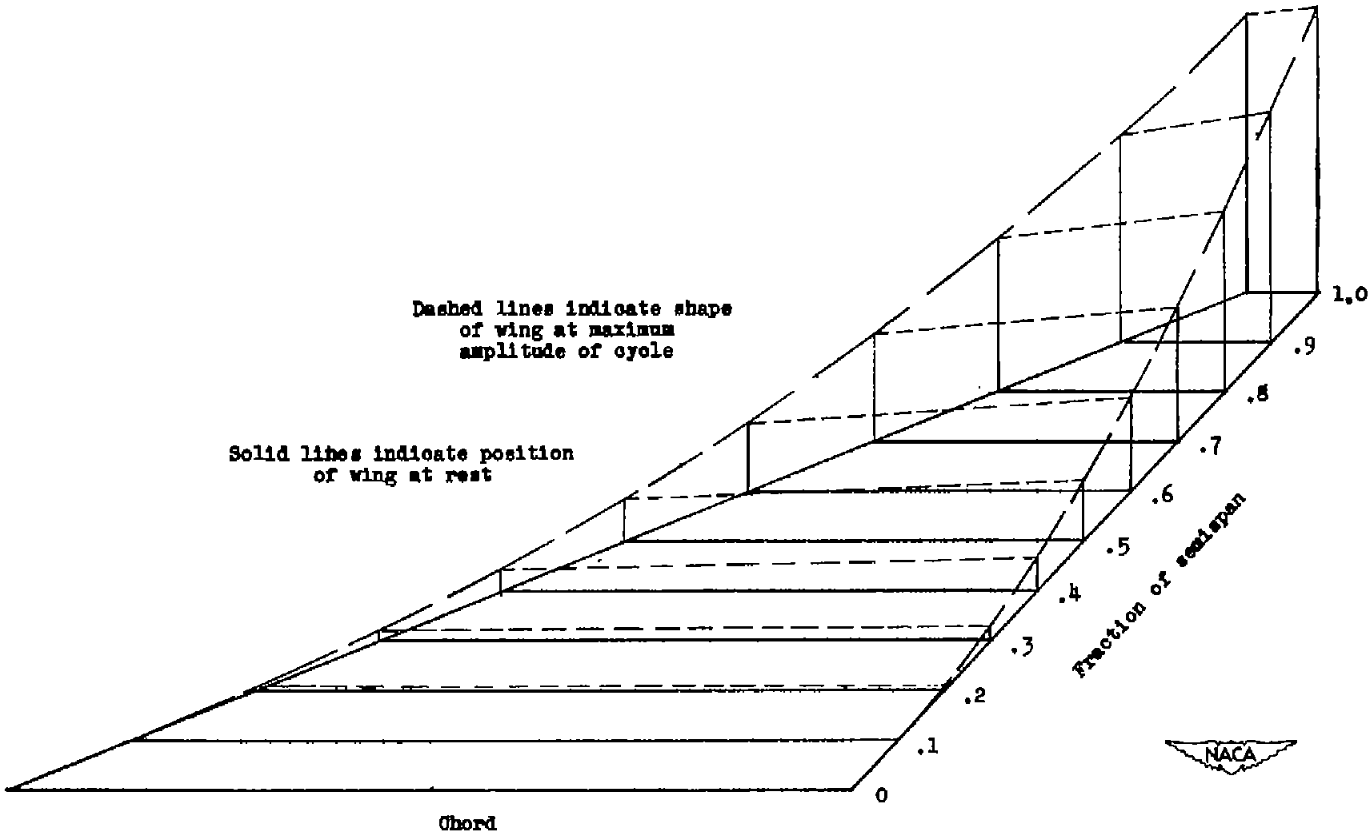
(d) Fourth natural vibration mode, ω_4 = 710 radians per second.

Figure 6.- Continued.



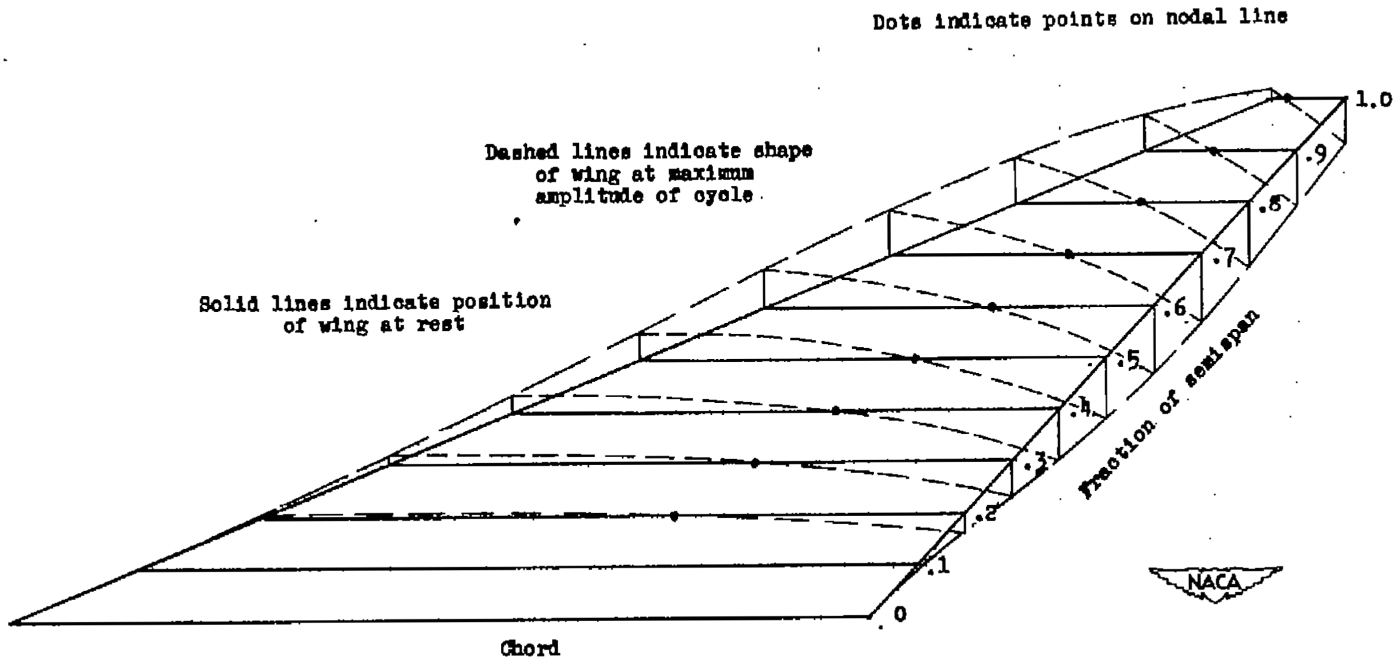
(e) Fifth natural vibration mode, $\omega_5 = 867$ radians per second.

Figure 6.- Concluded.



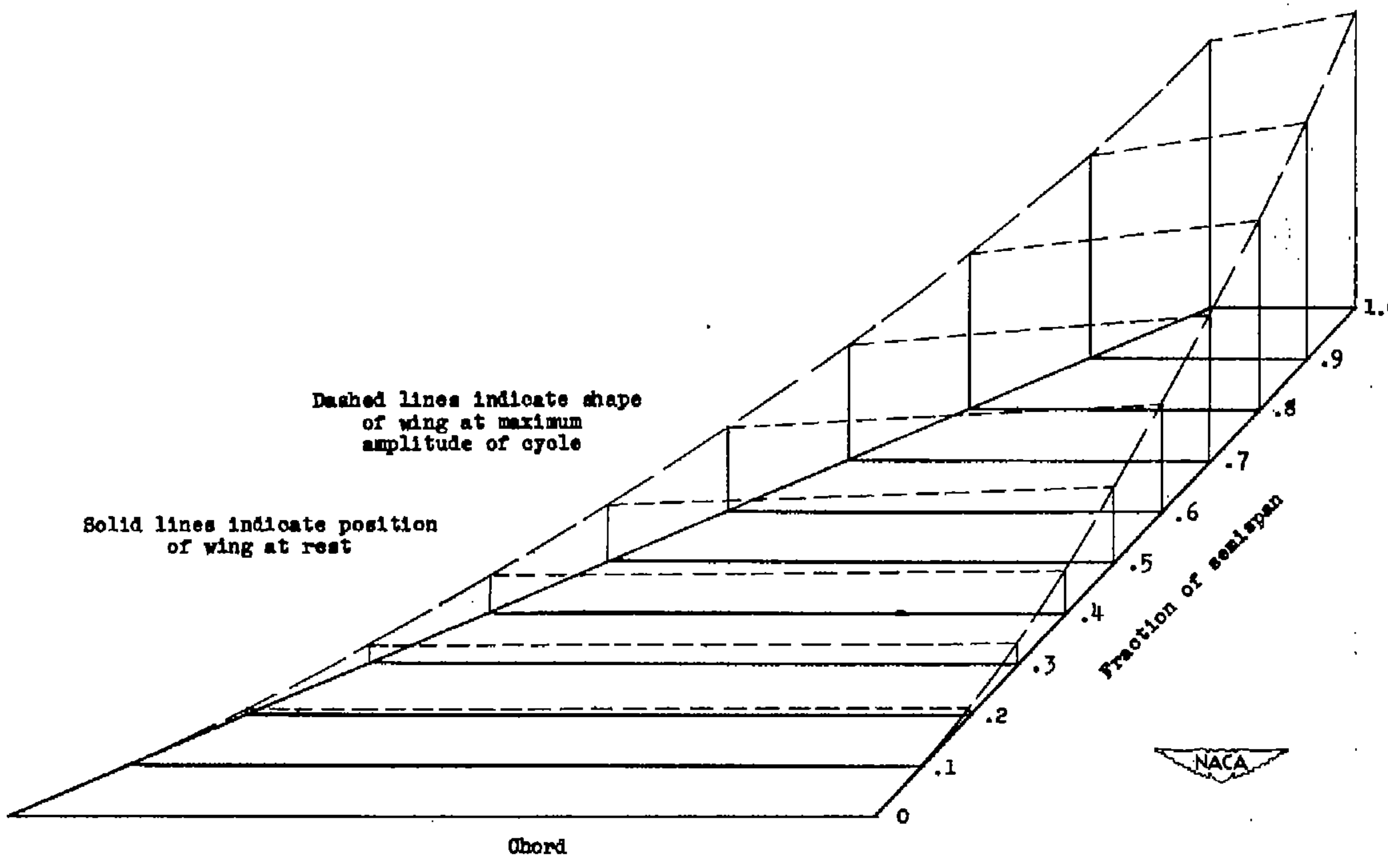
(a) First natural vibration mode, ω_1 = 179 radians per second.

Figure 7.- Natural vibration modes of model Ib.



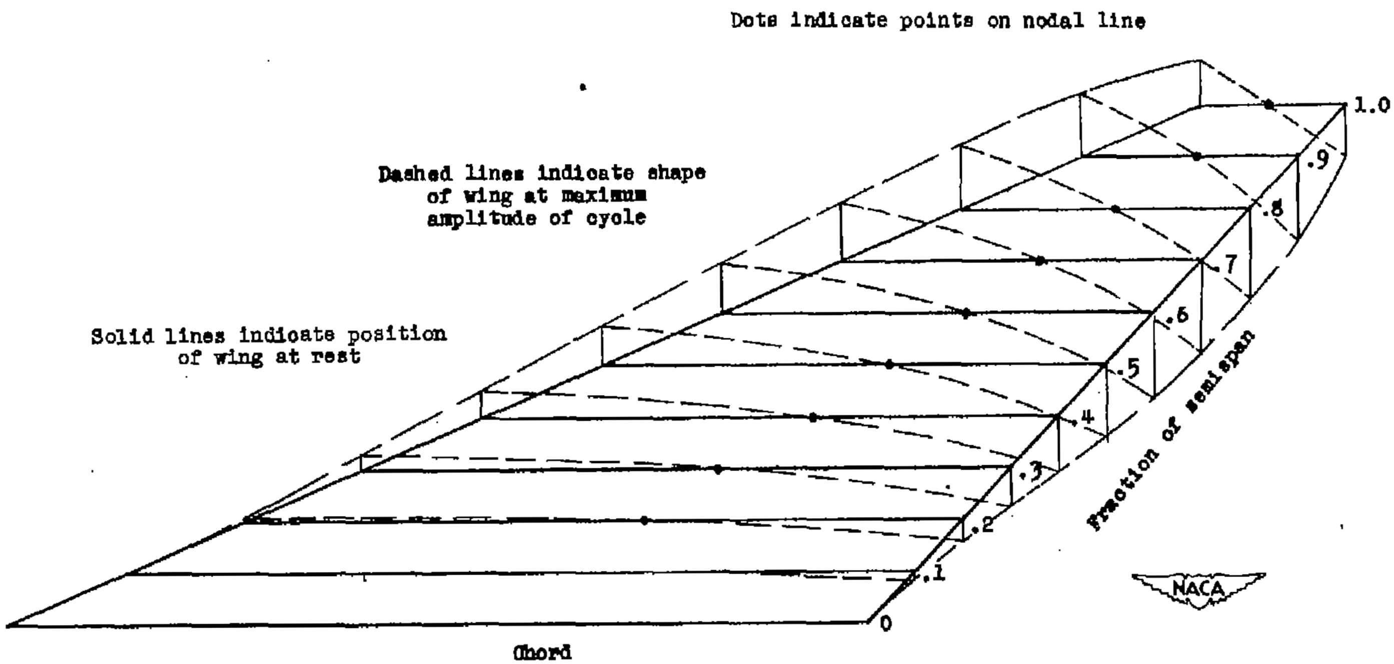
(b) Second natural vibration mode, $\omega_2 = 358$ radians per second.

Figure 7.- Concluded.



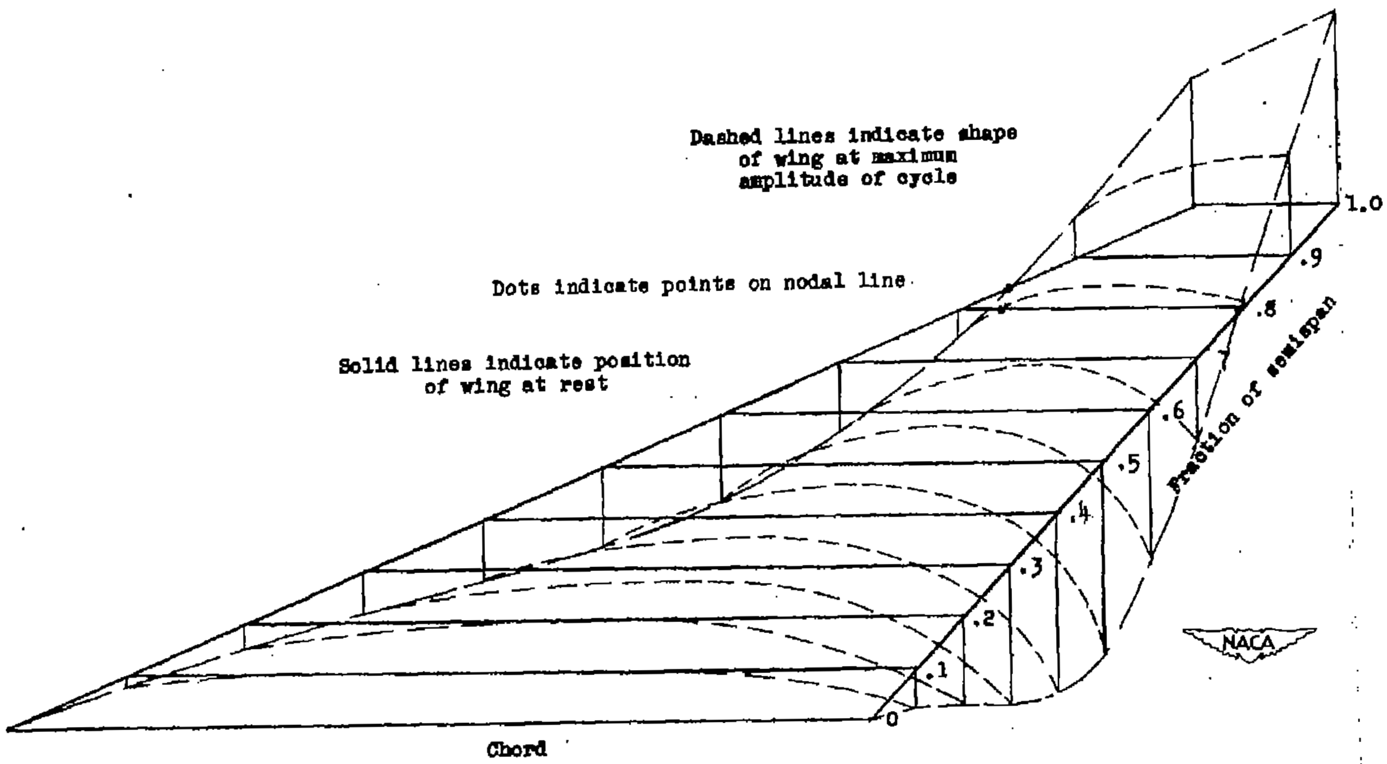
(a) First natural vibration mode, $\omega_1 = 204$ radians per second.

Figure 8.- Natural vibration modes of model Ic.



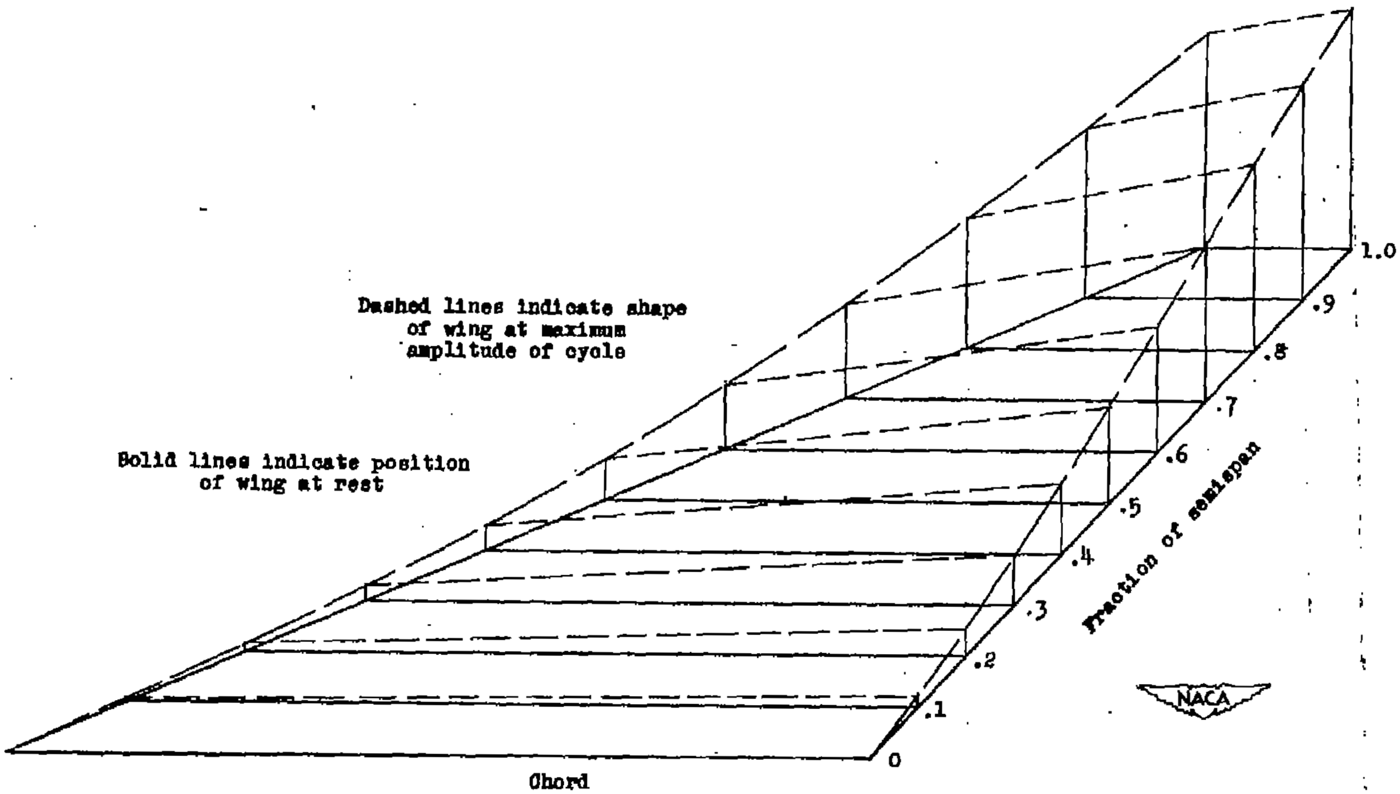
(b) Second natural vibration mode, $\omega_2 = 364$ radians per second.

Figure 8.- Continued.



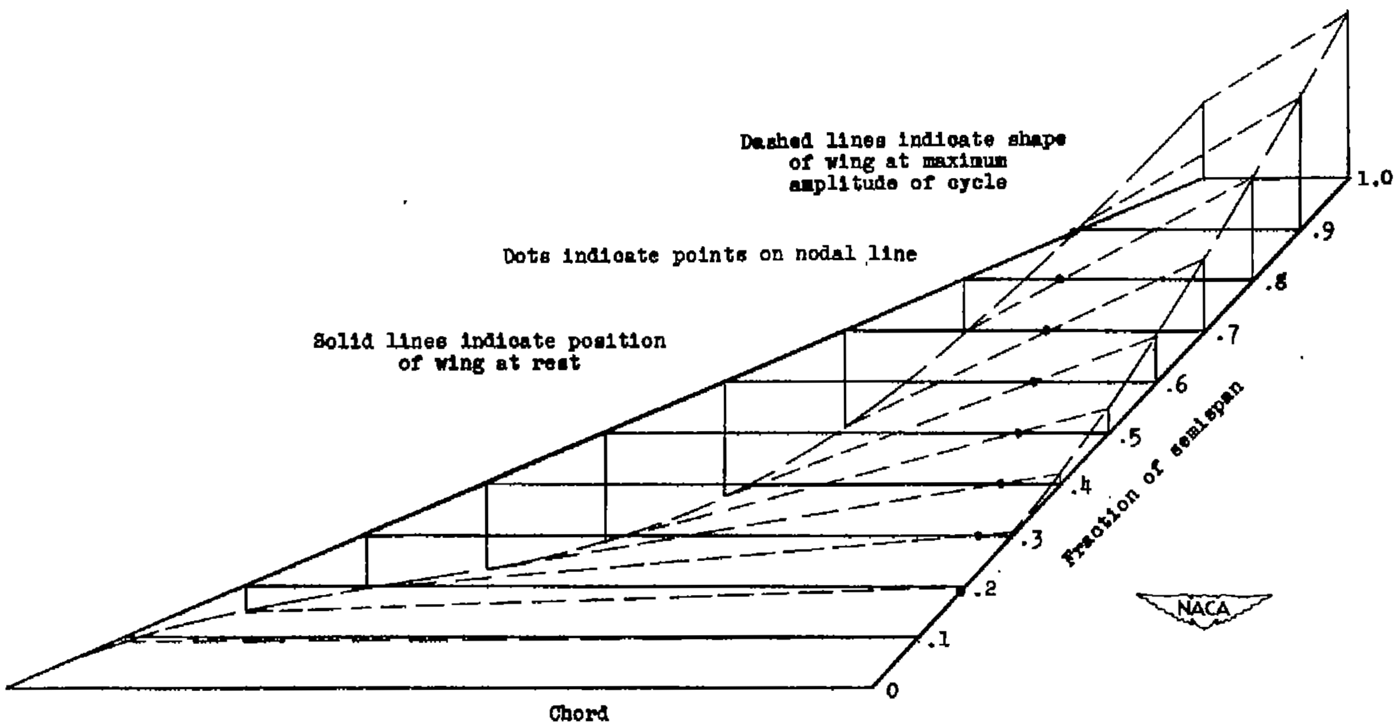
(c) Third natural vibration mode, $\omega_3 = 521$ radians per second.

Figure 8.- Concluded.



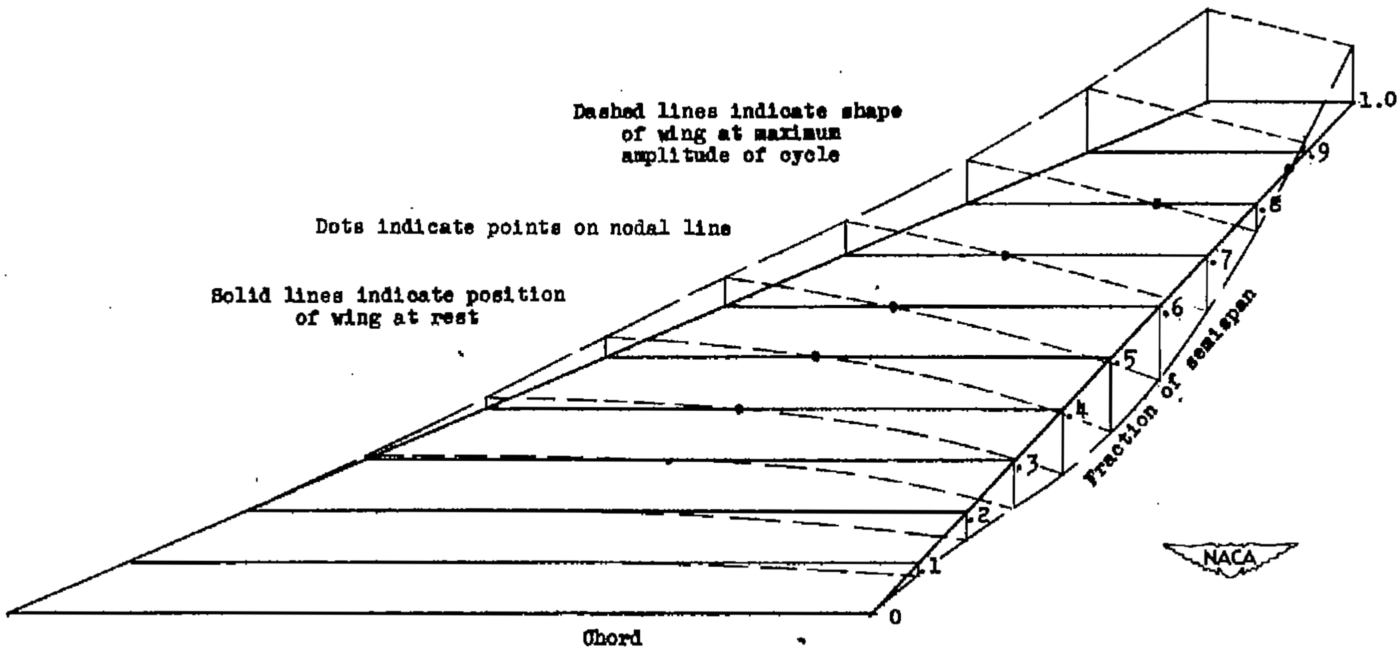
(a) First natural vibration mode, $\omega_1 = 28.3$ radians per second.

Figure 9.- Natural vibration modes of model IIIc.



(b) Second natural vibration mode, $\omega_2 = 126$ radians per second.

Figure 9.- Continued.



(c) Third natural vibration mode, ω_3 = 176 radians per second.

Figure 9.- Concluded.

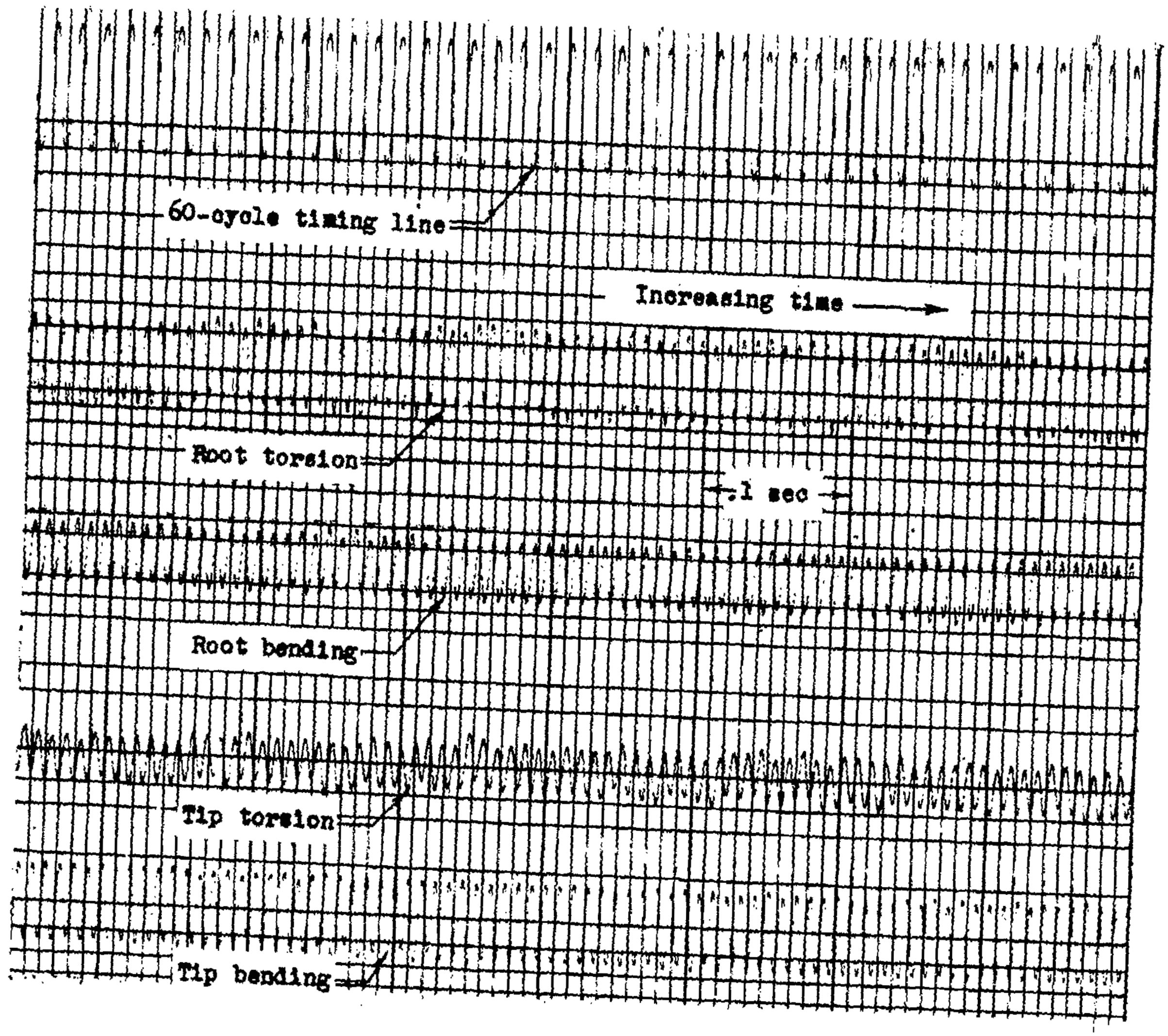


Figure 10.- Sample oscillograph record. Model Ib at flutter.

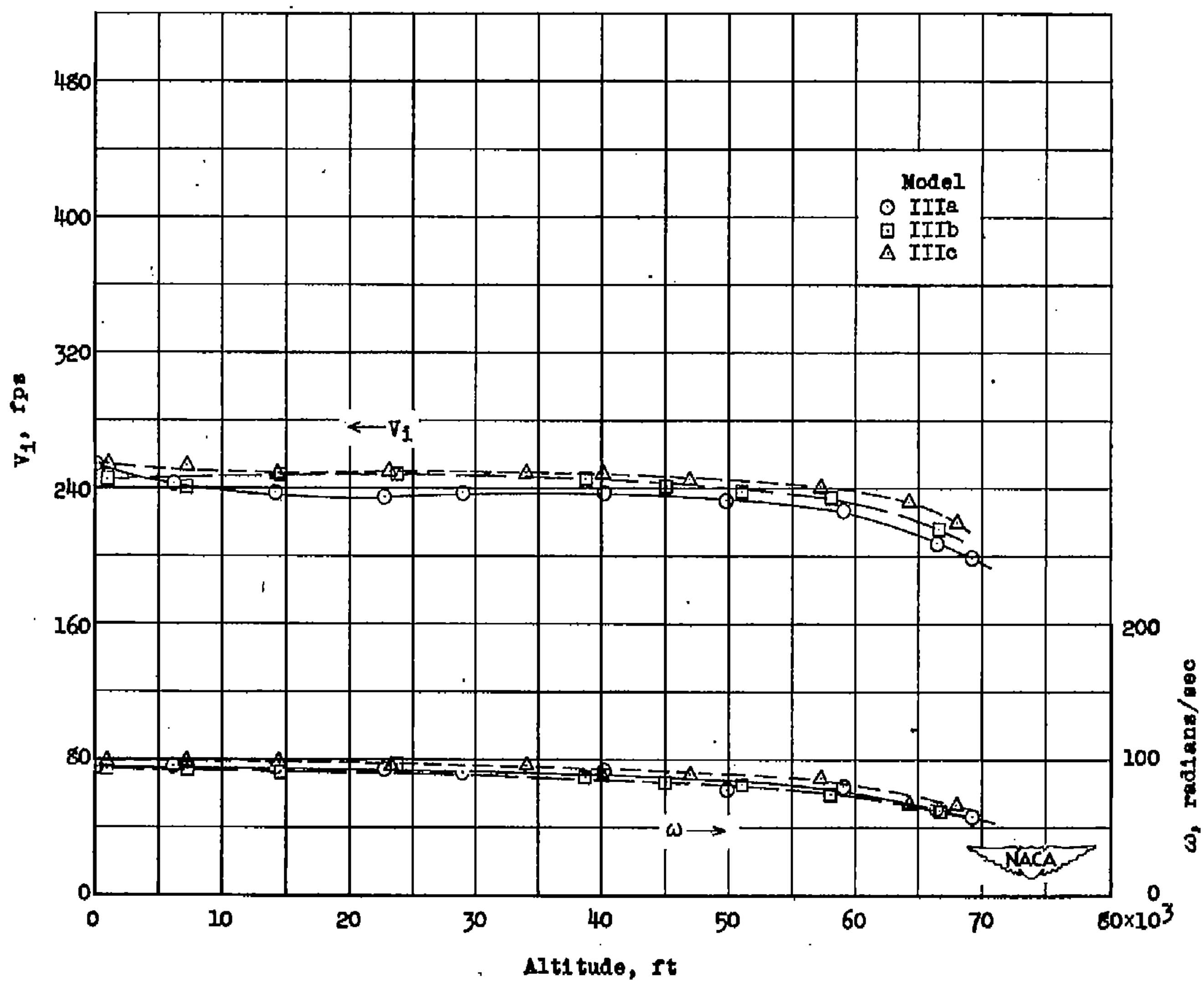


Figure 11.- Variation of indicated flutter velocity and circular flutter frequency with altitude; model III.

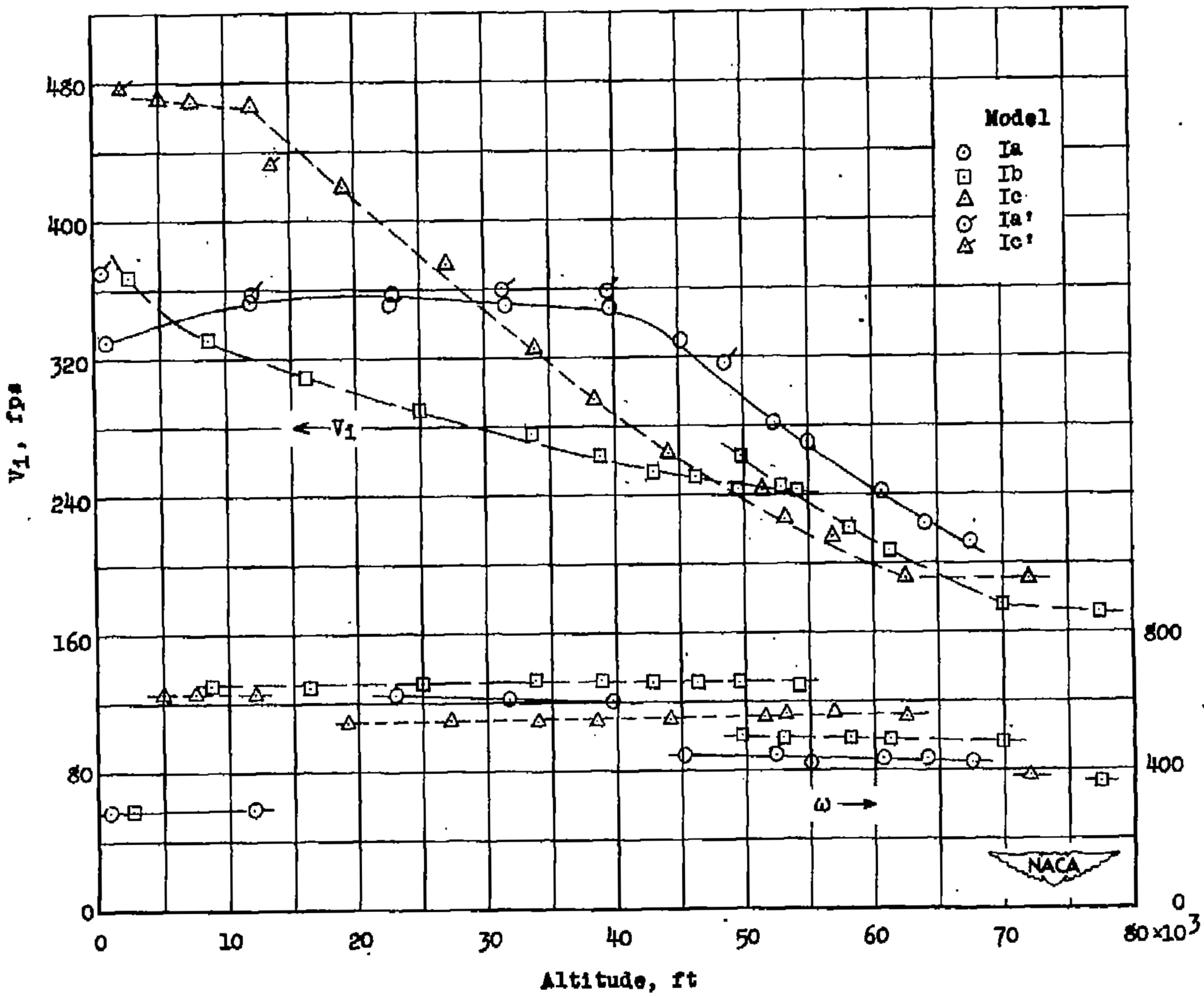


Figure 12.- Variation of indicated flutter velocity and circular flutter frequency with altitude; model I.

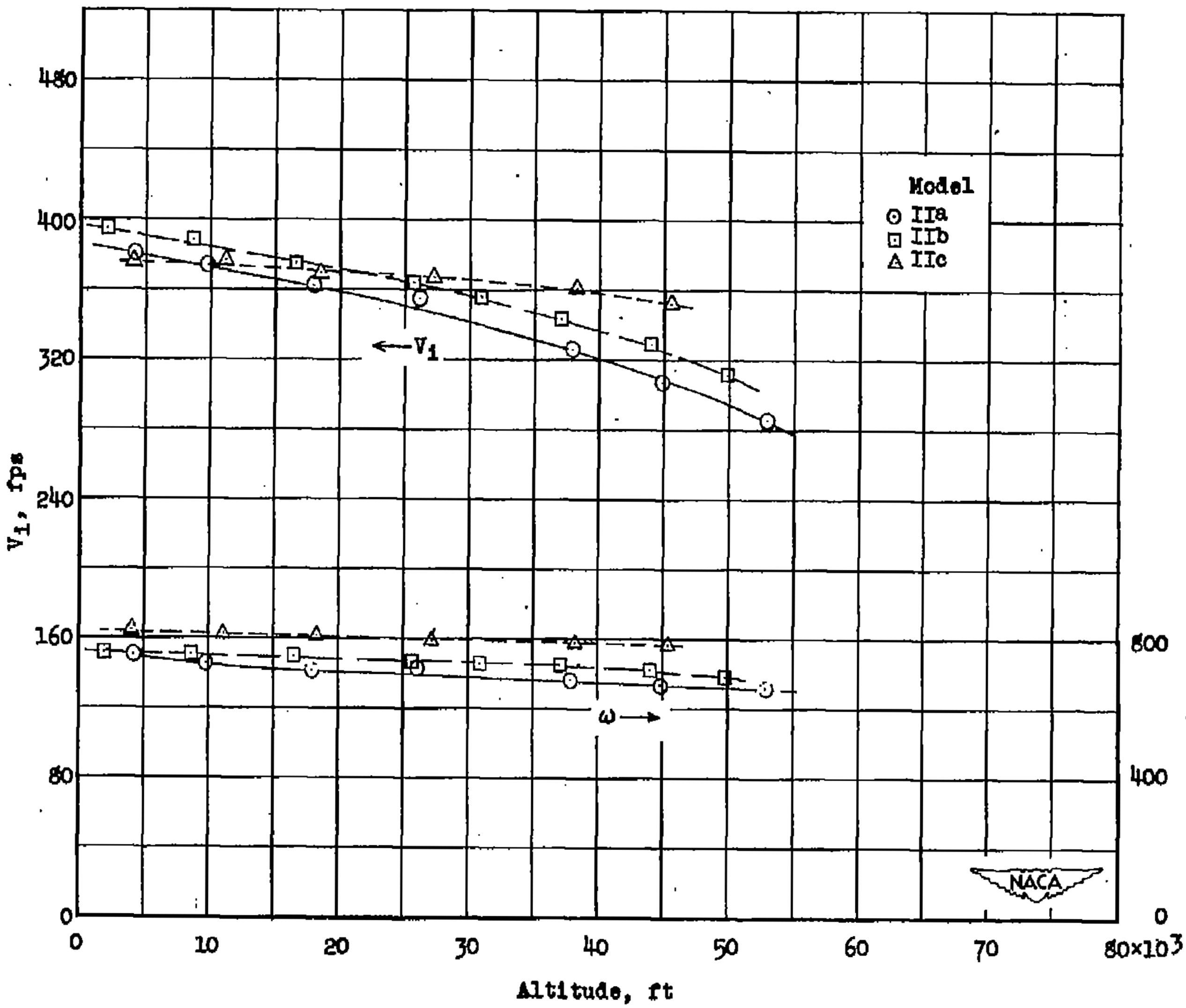


Figure 13.- Variation of indicated flutter velocity and circular flutter frequency with altitude; model II.